BACKPAPER EXAMINATION
M. Math II YEAR, I SEMSTER, 2016-17
FOURIER ANALYSIS

Max. marks: 100 Time limit: 3hrs

1. Let $T : \mathbb{R}^k \to \mathbb{R}^k$ be a linear isometry such that $\det(T) = 1$. Let $f \in L^1(\mathbb{R}^k)$ be such that $f(x) \equiv g(\|x\|)$ for some function g. Show that $\hat{f}(x) \equiv h(\|x\|)$ for some h. [10]

2. Let $\{\Phi_n\}$ be a approximate identity for $L^1([-\pi, \pi])$. If $f \in L^1([-\pi, \pi])$ and x is a Lebesgue point of f show that $(\Phi_n * f)(x) \to f(x)$. [10]

3. If f is a non-negative trigonometric polynomial show that there exists a trigonometric polynomial g such that $f \equiv |g|^2$. [25]

Hint: consider the roots of the polynomial $p(z) = z^N \sum\limits_{j=-N}^{N} c_j z^j$ where $c_j = \hat{f}(j)$.

4. Let $f(x) = \sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n^2}$. Show that the Hilbert transform g of f is given by
$g(x) = \frac{x^2}{4} - \frac{\pi^2}{12}$, $-\pi \leq x \leq \pi$. [25]

5.

a) Is there a constant c such that $c(I_{[0,\frac{1}{2})} - I_{[\frac{1}{2},]1})$ is scaling function? Justify. [10]

Hint: look at the Fourier transform at 0.

b) Does there exist a pair $\{a, b\}$ of positive real numbers such that $a < b$ and $xI_{(a,b)}(x)$ is a wavelet ?. Justify. [10]

6. Let $f \in L^1(\mathbb{R})$, g be a probability density function on $\mathbb{R}$ and $f = f * g$ a.e.. Prove that $\int f(x)dx = 0$. [10]